1. Introduction

Because we view the three-dimensional world from two horizontally separated eyes, each eye has a slightly different view of the environment. Wheatstone’s invention of the stereoscope showed that small differences between the images in the two eyes produced by viewing a three-dimensional scene from different vantage points allow recovery of the depth relationships that produced the differences. For many years the emphasis has been on what seemed like the simplest and most potent aspect of this information namely the horizontal differences in image width (horizontal disparity) produced by points at different depths. Since the invention of the random dot stereogram (Julesz) an accompanying preoccupation has been to model the processes by which the visual system decides which elements in the two eyes “correspond” in the sense of originating from the same configurations in the environment. Disparity can only be evaluated for corresponding images. Much more recently another aspect of stereoscopic vision has begun to receive attention. Another consequence of viewing a three dimensional scene from two laterally separated eyes is that surfaces and objects at different distances occlude one another to different extents in the two eyes, resulting in image points that are visible to one eye but not the other. These monocular features have no corresponding match in the other eye and therefore no binocular disparity can be calculated for these points. One might be tempted to view monocular features as uninformative noise that is tolerated by the stereoscopic system and perhaps allocated to a depth plane after depth is resolved binocularly. Some computational models of stereopsis, which will be discussed in Section 5, have attempted to incorporate monocular features in their recovery of depth. In the past 18 years, a number of findings have clearly indicated that monocular details contribute to binocular depth perception (for example: Gillam, Blackburn & Nakayama; Gillam & Borsting; Gillam & Nakayama; Grove, Gillam & Ono; Liu, Stevenson & Schor; Nakayama & Shimojo; Ono, Shimono & Shibuta; see also Howard and Rogers, 2002 for a full review).

In all of these studies the monocular elements are clearly distinct from the binocular regions. However there are binocular stimulus conditions that are ambiguous with respect to whether monocular regions exist or not. For example a
horizontal line with different widths in the two eyes could be either a slanted line in space with horizontal disparity of its endpoints or it could be a horizontal line in the frontal plane with one end differentially occluded in the two eyes by a nearer surface. In the latter case the two images of the end of the line where the occlusion occurs would be non-corresponding. These two possibilities are illustrated in Fig. 1. Fig. 1A shows a horizontal line slanted such that its left end is farther than the right projecting a wider image in the left eye than in the right. On the other hand, Fig. 1B shows differential occlusion of a frontoparallel line by a nearer surface on the right such that the right eye sees less of the far line than the left eye. In this case there is the same horizontal width difference between the left and right eye views as in Fig. 1A. What additional information is required to disambiguate this stimulus situation? This is the topic of the present paper. In this review we will summarize recent experiments we have carried out to investigate potential sources of information about the origin of a given horizontal width difference. In Section 2, we will describe the conditions under which the addition of an explicit or illusory occluder can bias an ambiguous horizontal width difference towards an occlusion resolution. In Section 3, we will describe global differences between the two images in line extents and orientation that arise ecologically at occlusion boundaries and how these patterns differ from those that arise as a result of global slant. In Section 4, we summarize a number of experiments in which we have evaluated the effectiveness of these patterns in signaling whether slant or occlusion has occurred and describe two experiments investigating the limiting conditions under which these sources of information are effective. We summarize our results and conclude in Section 5.

2. Explicit and illusory occluders

As already indicated, the horizontal width difference of a uniform rectangle is ambiguous with regard to whether the distal object is slanted or partially occluded. Local disparity computations at the end points of the line would predict slant. However, Häkkinen and Nyman and Grove, Ono and Kaneko reported that perceived slant of a homogenous rectangle, predicted from local horizontal disparity computations, was greatly reduced when a taller binocular occluder was placed so that it abutted the rectangle on the same side as the eye with the narrower image. This is the position a nearer occluder would have to occupy to account for the horizontal truncation of one image of the rectangle. When the taller rectangle was placed on the same side as the eye with the wider image, or when the disparate rectangle was viewed in isolation, slant was perceived as predicted from geometry (Ogle).

Consider Fig. 2A in which four rectangles are presented to each eye such that, when fused,
the upper pair appear to slant with their inner edges nearer than the outer edges and the lower pair slant by an equal amount in the opposite direction. In Fig. 2B, a central occluder is added in the region between the left and right rectangles and an asymmetrical perception results. Now the slant of the upper rectangles has been dramatically reduced and they appear as a single flat horizontal rectangle, partially occluded by the grey vertical rectangle. The slant of the lower pair of rectangles remains unchanged. The outer edges still appear nearer than the inner edges abutting the central occluder. This asymmetry highlights the role of non-local information in the resolution of local horizontal width differences.

The above observations can be extended to the case where there is no explicit occluder present. The disparate black rectangles in Fig. 2C are identical to those in A. In C however, they are accompanied by four 3/4 Kaniza circles which induce an illusory occluder in the position of the explicit occluder in B. Here again, the asymmetrical perception is observed. The upper rectangles appear flat and partially occluded by the illusory occluder while the slant of the lower rectangles is unaffected.

Grove et al.\textsuperscript{13) confirmed the above observations in a series of three formal experiments. In the first, 16 of 17 observers responding to stimuli similar to those depicted in Fig. 2A reported slant in both the upper and lower pairs of rectangles. Only 2 of the 17 observers reported any slant in the upper pair of rectangles when responding to stimuli similar to Figure 2B. In two more experiments using multiple staircases and the method of adjustment, respectively, observers estimated the slant of the upper rectangles in Fig. 2B and C as nearly frontoparallel, while their estimates of the lower pair were equal to values predicted from local disparity computations.

As shown in Fig. 1B, to be consistent with occlusion, the occluding contour must be on the

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\textsuperscript{1}Grove, Ono & Kaneko\textsuperscript{13) noted one naïve observer who perceived an illusory occluder and no slant when responding to stimuli similar to the top rectangles in Figure 1A.
same side of the line/rectangle as the eye that sees horizontally less of it. Therefore, when the horizontal extent of the right eye’s image is less than the left eye’s, the occluding contour must be along the right side of the disparate line or rectangle. The contour must be on the left of the line when the left eye’s image is horizontally smaller than the right eye’s. This can be observed in Fig. 2B. When cross-fused the spatial relationship between the top pair of rectangles and the central occluder meets the constraint and the rectangles appear frontoparallel. This relationship is reversed for the bottom pair of rectangles such that the left eye’s image of the right rectangle is wider than the right eye’s image and the right eye’s image of the left rectangle is wider than the left eye’s. This inconsistent with occlusion, and the lower rectangles appear to slant with their outer edges near.

An appropriately placed occluder, explicit or illusory, dramatically biases the perceptual output of local computations with respect to width differences. Indeed what is entailed is a change from a response to an entire rectangle as having corresponding disparate images in the two eyes to a response in which only part of the image of the wider rectangle is seen as corresponding to the narrower image in the other eye. The additional width seen is interpreted as a monocular region and occluded in one eye.

3. Patterns of horizontal and vertical image differences

In Section 2 we established that a visible (explicit or illusory) and appropriately positioned occluder biases the interpretation of an ambiguous horizontal width difference towards an occlusion resolution rather than slant. In this section we will show that other non-local stereoscopic sources of information along an occlusion boundary carry potential information as to whether the distal object is slanted or partially occluded. Below we describe three such sources of information and in Section 4 we review experiments that evaluate the relative effectiveness of these factors.

3.1 Constant vertical position differences

When oblique lines in the frontoparallel plane are partially occluded by a nearer vertical contour, vertical differences exist in the positions of the end points of the lines along the occluding contour (Anderson15). This is illustrated in Fig. 3A. Depicted here are two parallel oblique lines that are partially occluded by a nearer vertical contour on the right, indicated by the vertical dashed line. More of the oblique lines are visible to the left eye than the right eye so that the right ends of the lines are in different vertical positions in the two eyes’ images. The dashed horizontal grey lines indicate the vertical position of the right end points of the lines in the left eye’s image and the dashed black lines indicate their vertical position in the right eye. Note, however, that since the two lines are parallel, the separation between the end points is identical in the two eyes’. Therefore the vertical difference of the endpoints is constant in magnitude and direction for the top and bottom lines, hence we shall refer to this information as constant vertical differences. Note that Grove, Byrne and Gillam16) originally referred to these as constant vertical disparities. Here we revise this term to avoid confusion with other well-documented forms of vertical disparity (see Section 5 for a discussion).

Since a deletion of the right ends of the lines in the right eye’s image is geometrically consistent with an occluding surface along the right side of the lines, we refer to this as a valid
occlusion condition. Reversing the two eyes' views renders the stimulus geometrically inconsistent with an occluding surface along the right side of the lines since the eye on the opposite side from the occlusion now has the wider image and we therefore refer to this as an invalid occlusion condition.

3.2 Variable vertical position differences

A second possible source of disambiguating information is illustrated in Fig. 3B. Here, the two lines have the same absolute orientation as the parallel lines illustrated in Fig. 3A. The only difference is that the bottom line has been tilted with the opposite sign as the top one. Though the vertical differences at the right ends have equal absolute magnitudes, an additional source of useful information may be in the separation of their right endpoints which is greater in the left eye than the right eye. This size difference, which we refer to as variable vertical differences, is another possible source of information that the horizontal width differences are due to occlusion rather than slant.

As in Fig. 3A, the occluding contour must be on the same side of the lines as the eye which sees the smaller image (the right side of the lines). Therefore, reversing the two eyes' views renders the stimulus geometrically inconsistent with an occluding surface on the right.

Fig. 3C shows the same horizontal width differences for oblique lines that would be produced by slant and by differential occlusion. Slant results in a horizontal magnification of one eye’s image relative to the other. The transformations resulting from slant differ in two ways from those produced by differential occlusion. First unlike differential occlusion, uniocular horizontal magnification does not produce vertical differences in the end positions of the left and right images of the lines. Secondly uniocular horizontal magnification produces an
orientation disparity between the left and right eye images which is not present in the case of differential occlusion. In theory, these differences between the effects of slant and differential occlusion on oblique lines (vertical positional difference at the endpoints and no orientation disparity in the occlusion case; no vertical positional difference at the endpoints and orientation disparity in the slant case) could disambiguate the horizontal width difference.

3.3 Relative horizontal line length

A third possible source of information indicating occlusion is available when individual lines in a set have different lengths. We refer to this cue as **relative line length**. As discussed above a single horizontal line or rectangle with a difference in width between its images in the two eyes, is completely ambiguous with respect to whether the width difference is due to differential occlusion or slant. However, the pattern of horizontal disparities across multiple horizontal lines of different length is different for occlusion and for slant. Occlusion will result in the deletion of an equal linear amount from one image relative to the other regardless of the line length. This is illustrated in Fig. 3D for two lines, differing in length by a factor of two. In this case an equal linear amount has been deleted from the aligned end of the right eye’s image. Global slant of the same two lines however will result in an additional length on each eye’s image which is proportional to its length as shown in Fig. 3E. In theory, if the horizontal width differences were a constant linear amount along a set of lines (vertically aligned on one end) an occlusion would be indicated. If the differences are proportional to the length of each line slant should be indicated. Obviously these two possibilities can only be distinguished when lines have different lengths. Although proportional differences in length are inconsistent with occlusion it should be noted that constant differences among all lines are always consistent with slant as well as occlusion. However, the slant would have to be different for each line length since an additional constant length is a different proportion for each. This is not preferred when conditions are valid for occlusion. Nevertheless, when the images are reversed between the eyes the images are rendered inconsistent with occlusion at the aligned ends and multiple slants are indeed seen as a response to a constant additional length on all lines (see Section 4.1).

In Fig. 4 we illustrate the factors and effects just described using stimuli presented in experiments to be described in the next section. Cross fusion of the right two panels will produce width differences consistent with valid occlusion. Cross fusion of the center two panels will produce width differences invalid for occlusion. Cross fusion of the left two panels will produce image differences consistent with slant. Fig. 4A illustrates the effect of constant vertical differences at the alignment. Fig. 4B illustrates the effect of variable vertical differences. Fig. 4C illustrates the effects of relative line length and Fig. 4D is a control with neither vertical differences of any kind nor differential line length. Note the strong illusory contour in the valid occlusion conditions of Fig. 4B and 4C indicating an occlusion resolution. Note the differential slants of the invalid pair in Fig. 4C where the constant differences among lines cannot be attributed to occlusion and are interpreted locally as width differences for each line.

4. Experiments

Having identified three possible sources of information that could distinguish slant from occlusion, we set out to determine their relative
effectiveness. In our first set of experiments (Grove et al.\textsuperscript{16}), we employed two criteria to indicate that occlusion was perceived. First was the absence of slant in the lines. Second was the perception of an illusory occluding surface with an illusory contour in depth along the aligned edge of the stimulus lines.

4.1 Patterns of vertical differences and horizontal disparity

Our first experiment measured the perceived
strength of the illusory contour in depth using a magnitude estimation procedure. The stimuli were stereograms of the 12 pairs illustrated in Fig. 4. Observers rated, on a scale of 0–10, the degree to which the stimulus lines appeared flat and an illusory contour appeared in depth along the aligned ends of the lines. To do this, we employed a modulus, which we called the full cue stimulus, (see Fig. 5) against which the three configurations plus an additional control configuration (Fig. 4D) were compared. Observers were to rate illusory contours generated by the test stimuli as 10/10 if they appeared as clear and stable as the modulus; rate them 0 if no illusory contour was visible; or intermediate values for percepts that were less vivid or less stable than the modulus. All test and control line configurations were presented in valid/invalid occlusion, and global slant stereo permutations.

The highest ratings for the illusory contour in depth, indicating that occlusion and no slant was seen, were observed for valid occlusion only. Ratings for the invalid and slant stereo permutations were all near zero. This is consistent with our analysis and predictions above. Furthermore, the ratings for valid occlusion versions of the variable vertical difference and relative line length stimuli were significantly higher than those of the constant vertical difference and control stimuli.

In the second experiment of our study, we measured the perceived depth interval between the illusory contour, if one was visible, at the aligned ends of the lines. We reasoned that if, in the case of valid occlusion, the horizontal disparity at the aligned ends of the lines is interpreted as resulting from an occluding contour, this disparity should be responded to as a depth difference between the illusory contour and the ends of the lines that is equal in magnitude to a depth difference defined by a conventional disparity. Indeed, this is what we found for valid occlusion versions of the full cue, variable vertical difference, and relative line length stimuli. However, depth estimates were significantly smaller for valid occlusion versions of the constant vertical difference stimulus. Depth estimates for all line configurations were little different from zero in the remaining stereoscopic conditions (invalid occlusion and global slant), indicating that an illusory contour was not seen in these conditions and that the lines themselves appeared slanted in depth.

In a third experiment, we measured the proportion of presentation time for which an illusory contour in depth was visible for valid occlusion versions of each of the five line configurations. Here we reasoned that the stability and persistence of an illusory contour in depth, indicating an occlusion resolution, might vary as a function of what type of information was present along the occluding contour. Perception times for the illusory contour in depth were significantly longer in the full cue, variable vertical difference and relative line length stimuli than in the constant vertical difference or control stimuli, complementing the rating and depth matching results of the
previous two experiments.

In summary, the superior ratings, depth settings and perceptual stability were observed for the variable vertical difference and relative line length stimuli. In both of these configurations, binocular image differences vary along the length of the occluding contour. The superior responses to these configurations over the constant vertical difference and control stimuli provide strong evidence that global information, such as variations in vertical or horizontal image differences along the contour, are more effective at eliciting a clear and stable occlusion resolution than image differences of equivalent magnitude that remain constant along the length of the contour.

Thus far we inferred occlusion and slant resolutions of the lines based upon indirect responses about the visibility of the illusory contour which accompanied an occlusion resolution but not slant. Gillam and Grove investigated the relative line length stimulus more closely than we did in the previous three experiments. We used a stereoscopic depth probe to measure the depth at each end of a short line and a line twice as long in a set of 10 horizontal lines of different length in four stereoscopic conditions expected to produce either slant or occlusion (see Fig. 4C for examples). The measured depths at the line ends were very close to the depths predicted on the basis of global information present.

If occlusion was seen, as predicted in the valid occlusion permutation, the lines should appear flat with all the depth attributed to a step between the lines and an illusory occluder at the aligned ends. Thus the depth difference between the line ends should be small regardless of line length and this is what we found. For the invalid occlusion permutation we expected that each line would have a different slant. If the difference in depth settings between the left and right ends of the lines were the same for both the short and long line, the slant of the lines must necessarily differ by a factor or two because the proportional magnification for the short line would be double that for the long line. This is what we found. Conversely, if these same two lines had the same slant, the depth differences between the left and right ends of the lines would necessarily differ by a factor of two, owing to their different lengths. This is what we found for both positive and negative slants of the set of lines.

Gillam and Grove’s data provide further evidence that the visual system can use global information to distinguish a slant situation from an occlusion situation. In cases that are not consistent with global occlusion, precise slant is seen in accordance with local width disparities. Therefore, we concluded that local stereoscopic responses to disparities of individual line ends are overridden in favor of a more global occlusion solution when one is viable. Slant is the default interpretation when global occlusion is impossible (see Gillam and Grove for a detailed analysis of the matching process for these stimuli). In the same study, we extended these findings to show that more complex occlusion solutions can be elicited if, instead of a vertical aligned edge, the lines were aligned along a tilted or curved contour. Again a global occlusion solution was preferred in the valid occlusion permutation over multiple slanted lines, yet lines of multiple slants were seen in the invalid permutation. Interestingly, the visual system is capable of a high degree of sophistication in resolving sets of binocular disparities as inclined or curved occluding surfaces.

4.2 Limiting conditions

While the above experiments employed
stimuli consisting of ten lines, the contribution of global information in resolving ambiguous width differences probably depends upon the number of lines present as well as the smoothness of the contour along the aligned ends. Using the subjective rating method described above, Grove and Gillam\textsuperscript{18} explored the minimum conditions required to disambiguate slant and occlusion in the relative line length stimulus. We found that reducing the number of lines from 10 to 4 in a configuration of fixed height virtually eliminates the perception of an occluding contour. In this case, the separation between the lines increases as the number of lines decreases. In a second condition where the separation of lines did not increase with fewer lines, a vivid illusory contour in depth was still visible when just two lines were present. The fall off in illusory contour strength as line separation increases indicates a limited area (~1.12 degrees) within which information from two or more lines can be integrated to determine whether or not occlusion has taken place.

A second experiment in Grove and Gillam's\textsuperscript{18} study measured the tolerance for horizontal jitter along the aligned ends of the relative line length configuration consisting of 10 lines. Jitter values ranged between 0 and 24.5 min arc. Preference for an occlusion solution over lines of multiple slants was robust to random horizontal jitter of the individual lines up to about 5 min arc. Random horizontal jitter greater than 5 min arc degraded the illusory contour in depth such that patches of contour disappeared and some lines appeared slanted. While Gillam and Grove\textsuperscript{17} demonstrated that complex solutions along smooth occluding contours involving curved or inclined occluding surfaces with variations in alignment of up to 8 arc min, randomly jittered or jagged contours are far less likely to be interpreted as occlusions, even for moderate (>5 arc min) variations in alignment.

5. Discussion

Local horizontal width disparities are often ambiguous with respect to slant or occlusion along an occluding contour. We have shown, in the experiments reviewed here, that perceptual ambiguity is virtually eliminated in the presence of both explicit and illusory occluders. When appropriately placed, explicit or Kaniza-type occluders attenuate slant based on local computations and bias perception towards an occlusion resolution consistent with the global information present. Additionally, stereoscopic information exists in the global patterns of texture on the occluded object or surface which can disambiguate local horizontal width differences. Amazingly, this information is effective in disambiguating slant from occlusion in the absence of an explicit or Kaniza-type occluder. Instead, an illusory occluder is generated by the visual system to account for the occlusion resolution. We identified three sources of information available to the visual system along an occluding contour to disambiguate slant from occlusion. We named these sources constant vertical differences, variable vertical differences, and relative line length, respectively. In evaluating the relative contributions of these cues in resolving ambiguous local disparities, we have shown that variable vertical differences and relative line length are clearly superior to constant vertical differences. This superiority is evidence that global information, integrated across two or more lines, is more effective than local information in resolving the horizontal disparities in these stimuli.

It is also evident from the experiments outlined here that orientation disparity
accompanying horizontal magnification of oblique lines is a weak cue in resolving horizontal disparities. If the absence of orientation disparity was a salient cue for disambiguating horizontal disparity, the subjective ratings, depth estimates, and perception times for the constant vertical difference configuration should have been better. Furthermore, fusing the invalid occlusion stereograms in Fig. 3 and Fig. 5, it can be seen that the lines appear at various slants according to their local horizontal disparities. Orientation disparity is absent in this permutation, consistent with a frontoparallel orientation, yet slant is seen. Our data suggest that if the conditions for an occlusion resolution are absent, slant is the default resolution even though it is inconsistent with the absence of orientation disparity.

In this paper we have revised the terminology we first used in Grove et al.\textsuperscript{16} What we referred to as constant vertical disparity and differential vertical disparity in that paper, we now refer to as constant vertical differences and variable vertical differences, respectively, so as to avoid confusion with the conventional definition of vertical disparities which are known as a cue to distance and eccentricity (Mayhew and Longuet-Higgins\textsuperscript{19}; Gillam and Lawergren\textsuperscript{20}). The vertical image differences discussed here arise from occlusion at the endpoints of the lines and do not correspond to a single distal location. Anderson\textsuperscript{15} defined these image differences as vertical half occlusions. While this definition is consistent with the geometry of the situation and the perceptual outcome of Anderson’s stimuli and our variable vertical difference stimuli, it assumes a priori knowledge of the scene layout that is unknown to the visual system presented with two 2-D retinal images.

Our experimental results should be of interest to computational approaches to stereoscopic occlusion. For example, Egnal and Wildes\textsuperscript{21} describe and evaluate five computational algorithms that detect monocular occlusion zones and incorporate them into a fully matched stereoscopic representation. While the five algorithms all detect monocular occlusion zones with varying success in selected natural images, none of the methods would detect an occlusion event in any of our stimuli. Moreover, each computational method would resolve all the stereoscopic permutations of our stimuli as resulting from slant. This is due, in part, to the fact that the lines in our stimuli are continuous and uniform with no discrete monocular features to be discerned from analyzing the two eyes’ images. Additionally, as mentioned above, slant is a fully plausible solution, based on local matches, in all our stimuli. Differences with regard to slant or occlusion only become discernable when horizontal width differences or vertical image differences are integrated across two or more lines. Clearly, the psychophysical findings outlined here are yet to be addressed by computational models.

Our observations might also be of interest to engineers working on remedies for perceptual distortions in 3-D displays. Ohtsuka, Ishigure, Kanatsugu, Yoshida and Usui\textsuperscript{22} found that as objects, with simulated depth, entered a 3-D display from one side, observers saw those objects slanted about a vertical axis until the entire object was visible. Because the entire object was not visible to the observers, the left and right eyes’ images differed in horizontal extent. For example, if the object were a fish entering the display from the left side with uncrossed disparity, the right eye’s image would enter the picture first and the fish’s nose would be further to the right than the left eye’s image. In this case, the two eyes’ images differ in their
horizontal extent and the resulting percept is a fish slanting with its head away from the observer until the entire fish is in view when it then appeared as intended—frontoparallel and in depth. These authors remedied this problem by adding a “virtual picture frame” made up of a Julesz style random dot pattern with crossed disparity surrounding the display, replicating the stimulus situation in Section 2, enabling observers to perceive the intended depth. As a final conclusion we might suggest that an additional tactic to reduce image distortions of this type might be to add oblique features of irregular orientation and length to the objects in the display so that variable vertical differences and patterns of relative horizontal disparity, present as the objects enter and exit the display, would further reduce the likelihood of these image distortions.

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